

9210-203

Level 7 Postgraduate Diploma in Engineering

Computational mechanics using finite element method

GLA d`Y`DUdYf

You should have the following for this examination

- one answer book

No additional data is attached

General instructions

- This paper consists of **seven** questions
- Answer **five** questions
- Use large, clear labelled diagrams or sketches whenever these help your answers.

- 1 A three bar truss ABCD is pin-supported at B, C and D as shown in Figure Q1(a). It is subjected to a concentrated load of 50 N at A. All members have axial rigidity (AE) 10^6 N. It has been decided to analyze this truss using Finite Element Analysis.
- Write element stiffness matrices of each truss member in global coordinate system. (6 marks)
 - Write global stiffness matrix for the truss by assembling individual stiffness matrices. (4 marks)
 - Apply boundary conditions and find the nodal displacement at point A. (10 marks)

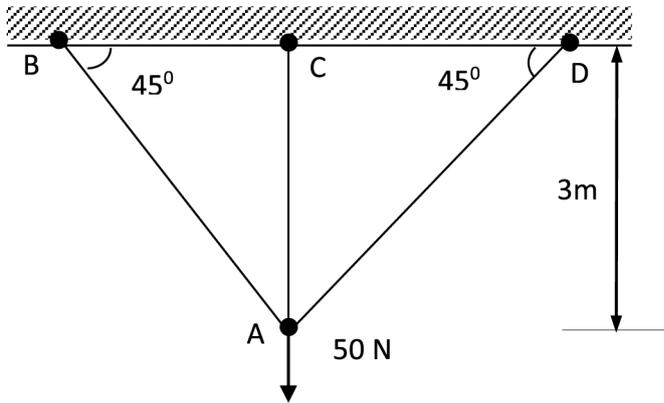


Figure Q1(a)

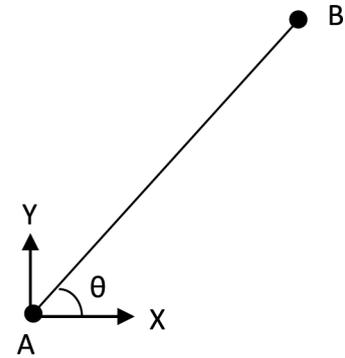


Figure Q1(b)

Hint:

For a pin-jointed truss member of length L , cross sectional area A and elastic modulus E (Figure Q1b), the relationship between nodal forces and displacements is written as, where ,

$$\begin{bmatrix} F_{ax} \\ F_{ay} \\ F_{bx} \\ F_{by} \end{bmatrix} = \frac{AE}{L} \begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix} \begin{bmatrix} U_{ax} \\ V_{ay} \\ U_{bx} \\ V_{by} \end{bmatrix}$$

where $C = \cos\theta$, $S = \sin\theta$

U = Horizontal nodal displacement

V = Vertical nodal displacement

F_{ax} = Horizontal nodal force

F_{ay} = Vertical nodal force

- 2 a) Describe the fundamental difference between the beam element and bar element used in Finite Element Analysis. (4 marks)
- b) A cantilever beam is subjected to two point loads as shown in Figure 2(a). It is required to analyze this beam using Finite element analysis using two beam elements AB and BC. Flexural rigidities of AB and BC are $3EI$ and $2EI$, respectively. You can consider EI as 25 kNm^2 .
- i) Write the individual stiffness matrices for elements AB and BC. (6 marks)
- ii) Using the results in (i), assemble the global stiffness matrix of the system. (4 marks)
- iii) Write load-displacement relationship for the system. (6 marks)

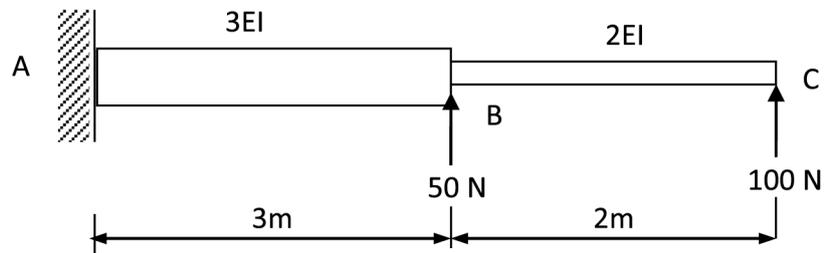


Figure Q2(a)

Hint:

The stiffness matrix for an elastic beam element of length L and flexural rigidity EI shown in Figure Q2(b) is given by,

$$\frac{EI}{L} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

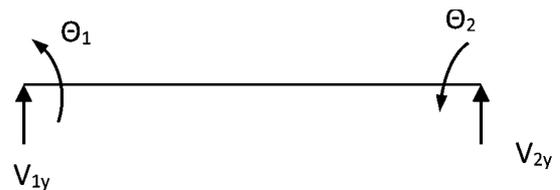


Figure Q2(b)

- 3 A continuous beam ABCD is subjected to a loading arrangement as shown in Figure Q3(a). Flexural rigidity of members is 25 kNm^2 and is supposed to be analyzed using Finite Element Analysis.
- Find the equivalent nodal forces and moments at four nodes of the beam. (6 marks)
 - Using force stiffness relationship, solve the structure. (14 marks)

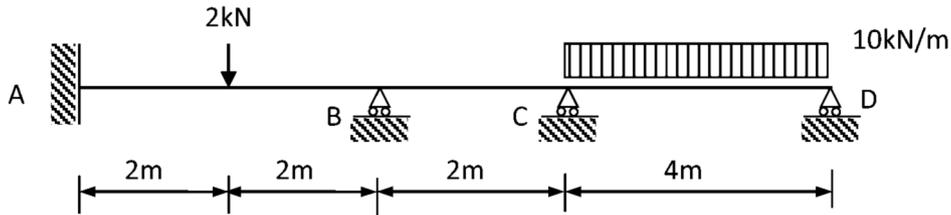


Figure Q3(a)

Hint:

For a given beam element, the equivalent nodal forces and moment due to a concentrated load P acting at the centre and uniformly distributed load of intensity W are given as

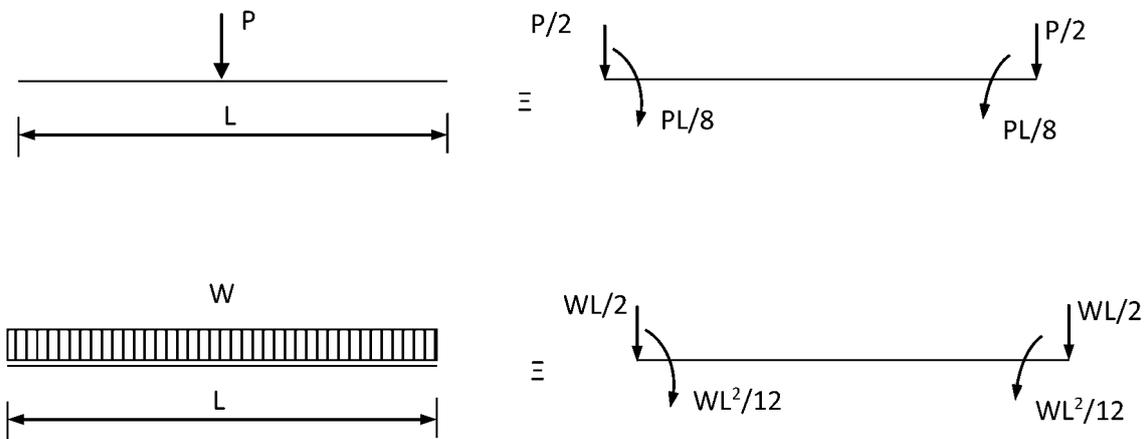


Figure Q3(b)

- 4
- Explain following technical terms used in Finite Element Analysis.
 - Discrete modeling. (2 marks)
 - Gauss Quadrature. (2 marks)
 - Gauss Siedel iteration technique. (2 marks)
 - Find the shape function for a beam element. (4 marks)
 - Describe properties of shape functions considering a beam element as an example. (5 marks)
 - Obtain the element stiffness matrix for a beam element. (5 marks)

- 5 a) What do you understand by 'Constant Strain Triangle' used in two dimensional finite element analysis? (4 marks)
- b) A steel plate of 4 mm thickness is subjected to in-plane loading of 2N/cm and supported by a pin and a roller support as shown in Figure Q5. Considering that the plate is consisting of two constant strain triangles,
- i) Write strain-displacement matrices. (4 marks)
- ii) Write stiffness matrix for the whole plate. (4 marks)
- iii) Determine nodal deflections. (8 marks)

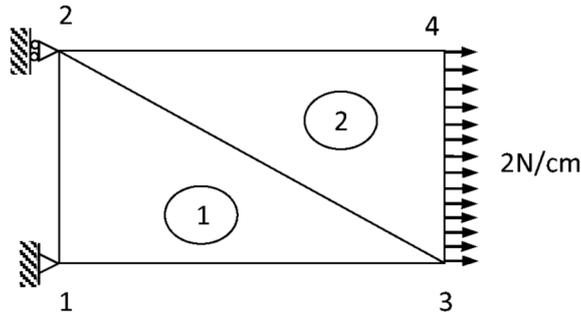


Figure Q5

- 6 a) What is meant by homogeneous isotropic materials? (4 marks)
- b) Draw stress-strain diagrams of following material idealizations and explain their characteristics.
- i) Linearly elastic perfectly plastic. (3 marks)
- ii) Perfectly plastic. (3 marks)
- iii) Liner elastic linear plastic. (3 marks)
- c) Describe why it is necessary to satisfy displacement compatibility relation in Finite Element Analysis. (7 marks)
- 7 a) Describe what is meant by validated finite element model. (6 marks)
- b) Describe different situations where three dimensional finite element modeling is required. (6 marks)
- c) Describe basic concepts of applying finite element modeling in dynamic analysis. (8 marks)