Level 3 Certificate in Using and Applying Mathematics

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Pre-release material

Sample assessment 2

This booklet will be issued to centres two months in advance of the date of examination.

Candidates will be issued with a clean copy of this booklet for each paper sitting. Copies will be issued at the start of the examination session and collected at the end of the session. Candidates must **not** take their own copies of this booklet into the examination.

Centres should ensure that candidates are familiarised with the contexts and information contained in this booklet in preparation for the examination.

All examination questions will be based on this material.

This booklet contains a set of four documents
1) Running a vehicle
2) Debtor days
3) Designing for comfort
4) Earnings.

1) Running a vehicle

There are many costs associated with running a vehicle. A major cost is that of fuel. This can and does vary during the year, often rising. The fuel efficiency of vehicles is therefore an important factor to take into account.

Almost all vehicles that are approved for sale in the European Union undergo tests to find their fuel efficiency. In the UK, vehicles are tested in laboratories to find out the number of miles per gallon (mpg) they will be expected to drive in different conditions (urban, extra-urban and combined) and this is quoted by manufacturers in advertising material.

Figure 1 shows an example of this data. The data includes figures for petrol and diesel models of a selection of vans. Although diesel vans are often more efficient than petrol vans they usually cost somewhere between £1000 and £2000 more to buy.

Model	Urban mpg	Extra-urban mpg	Combined mpg
Ford Fiesta (petrol)	40.9	67.3	55.4
Ford Fiesta (diesel)	64.2	85.6	76.0
Vauxhall Corsavan (petrol)	44.1	65.7	56.0
Vauxhall Corsavan (diesel)	52.3	74.3	64.4
Fiat Fiorini (petrol)	35.3	52.3	44.7
Fiat Fiorini (diesel)	55.4	74.3	65.8

Figure 1. Fuel efficiency of some small vans

Petrol and diesel is now sold in litres in the UK so when calculating driving costs for a journey motorists need to know that there are 4.546 litres in a gallon.

Another major financial consideration associated with running a vehicle is its depreciation in value. As soon as you drive a brand new vehicle away from the garage where you buy it the resale value will be considerably less than the price you paid for it. The AA's advice about vehicle depreciation is given in **Figure 2**.

Typically the rate of depreciation slows with age.

Year one A new vehicle loses value as soon as you drive off the forecourt and by the end of the first year will have lost around 40% of its value. This varies a lot though and the best may lose as little as 10%.

Year three If you do 10,000 miles a year, the average vehicle will have lost around 60% of its value by the end of its third year.

Figure 2. AA advice

2) Debtor days

Debtor days is a measure used by some finance offices to provide an indicator of how long customers take to settle their accounts.

The spreadsheet below shows how this is calculated for a company that hires and sells equipment to small businesses such as building companies. These companies are categorised as either being construction (CON) or engineering (ENG). Each sale is also allocated to one of the company's only two sales centres in Birmingham and Manchester.

Figure 1 calculates the *debtor days* measure for both sectors (construction and engineering) for the two sales centres (Birmingham and Manchester) as well as the company overall.

	A	B	C	D	E	F	G	H	I
1	Debtor days								
2									
3	Month			February					
4									
5									
6	Number of working weeks			11.5					
7									
8					CON	ENG	Birmingham	Manchester	Company overall
9				December	£31,556	£17,525	£25,461	£23,620	£49,081
10				January	£39,475	£21,564	£37,469	£23,570	£61,039
11				February	£68,945	£46,435	£61,834	£53,546	£115,380
12	3 month total				£139,976	£85,524	£124,764	£100,736	£225,500
13									
14	3 month total/working weeks				£12,172	£7,437	£10,849	£8,760	£19,609
15									
16	Total sales=(3 month total/working weeks)*52				£632,935	£386,717	£564,150	£455,502	£1,019,652
17									
18	DEBTORS				£124,528	£45,672	£96,856	£73,344	£170,200
19									
20	DEBTORS without sales tax				£103,773	£38,060	£80,713	£61,120	£141,833
21									
22	debtor days=(Debtors without sales tax/Total sales)*365				60	36	52	49	51
23									
24									

Figure 1. Spreadsheet used to calculate the indicator *debtor days*.

The data for each month is fed into the spreadsheet in columns E, F, G and H, rows 9, 10 and 11. The values in cells I9, I10 and I11 are calculated using these data.

To calculate the most recent value of *debtor days* the most recent three month's sales values are used. These are summed in row 12 to give the value '3 month total'. In row 14 this sum is divided by the number of working weeks in the three months. In row 16 this is multiplied by 52 (the number of weeks in the year) to give an estimate of "total sales for the year". This gives a sub-step on the way to calculating the final indicator.

Row 18, labelled as "DEBTORS", is data that is inserted: it gives the total outstanding debt for the sector or sales centre. This value includes sales tax which is deducted in row 20 so that values in this row give the amount "debtors without sales tax". Finally the measure *debtor days* is found in row 22 by calculating ("debtors without sales tax"/"total sales for the year") × 365.

This gives a measure of the average number of days it takes customers to pay their debts to the company.

In the specific example shown in **Figure 1** *debtor days* is calculated for February using the data for the months of December, January and February. This period includes Christmas so the number of weeks is taken to be 11½, so as to take into account breaks for the Christmas and New Year holidays.

Figure 2 shows the values of the measure *debtor days* calculated for February and each of the previous five months together with the simple moving average based on three months. The moving average is found for a particular month by finding the mean of the values for the month together with those for the previous two months. The moving average is used to smooth out fluctuations in the data (for example, caused by the holiday period over Christmas).

	A	B	C
1	**Debtor days - moving averages**		
2	**Company total**		
3	**Month**	**debtor days**	**moving average**
4	**September**	50	45
5	**October**	49	47
6	**November**	47	49
7	**December**	42	46
8	**January**	45	45
9	**February**	51	46
10			

Figure 2. Spreadsheet showing values calculated for the measure debtor days for each month together with the values of the moving average calculated each month based on values for three months.

3) Designing for comfort

Designers of many products have to ensure that they are comfortable for use by their target users. For example, chairs have to be designed to have a comfortable width, depth and height from the ground. This requires designers to draw on anthropometrical data (ie data about average dimensions of people).

For example, if a designer was designing a chair for use in a primary school, they would use anthropometrical data for young children, whereas if the chair was to be used in a cinema it would be important to use data for adults.

The two diagrams in **Figure 1** indicate some of the body dimensions that are typically measured. The Table in **Figure 2** (on the next page) gives values for the mean and standard deviation (SD) for each of these dimensions for men and women aged 18-65.

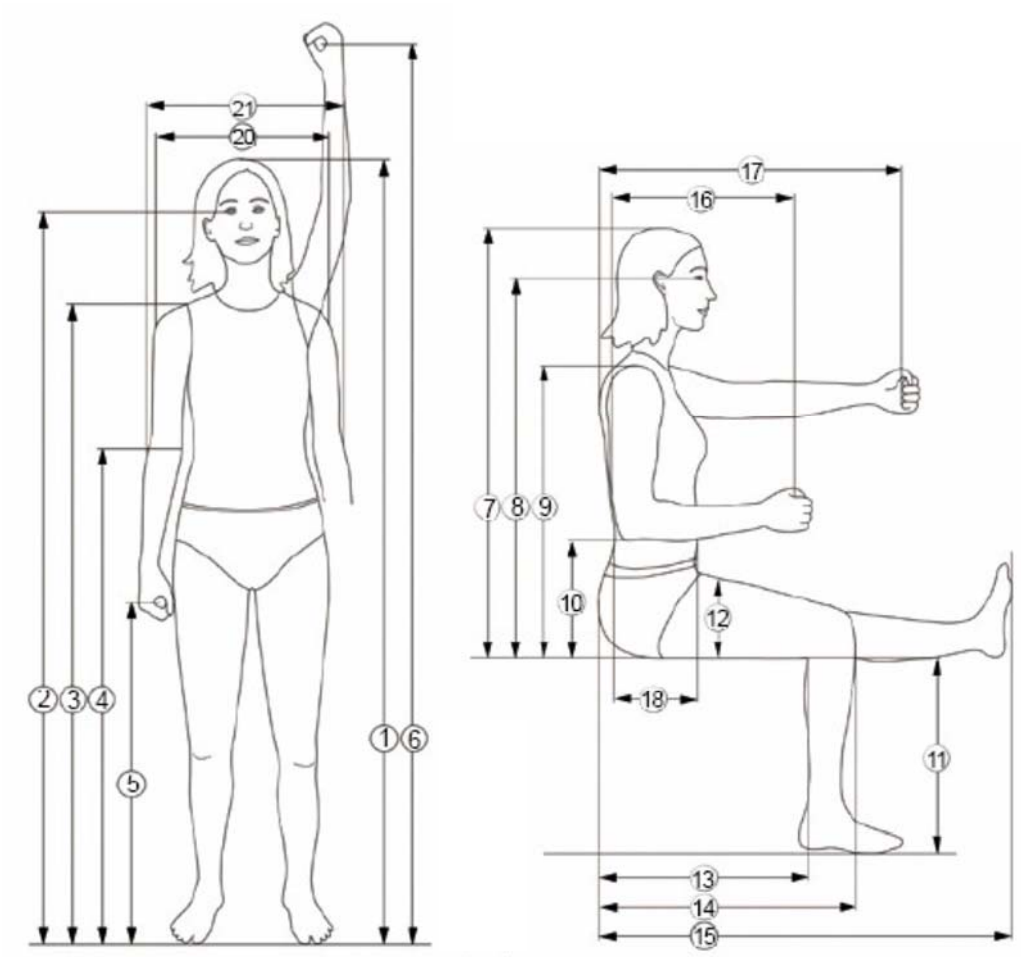

Figure 1. Some of the body dimensions that are typically measured.

		men		women	
	Dimension (in mm)	**mean**	**standard deviation**	**mean**	**standard deviation**
standing					
1	stature	1766	76	1646	68
2	eye height	1650	74	1537	65
3	shoulder height	1445	67	1342	60
4	elbow height	1132	52	1057	48
5	fist height	782	36	750	44
6	vertical grip reach	2090	107	1901	80
sitting					
7	crown-buttock height	919	36	867	33
8	eye height	804	35	760	31
9	shoulder height	611	31	577	28
10	elbow height	246	25	242	23
11	popliteal height	461	23	432	20
12	thigh thickness	150	15	149	12
13	buttock-popliteal length	504	25	488	24
14	buttock-knee length	619	31	594	28
15	buttock-heel length	1055	57	1061	54
16	elbow-fingertip length	353	18	316	16
17	forward grip reach	800	40	734	34
18	abdominal depth	243	32	231	28
19	hip breadth	369	26	404	34

Figure 2. Body dimension data (all values in mm).

For example, consider an art gallery designer who wants to hang all paintings so that the centres of each are a height from the floor so that they are within $\pm$ one standard deviation of mean eye height. The Table in **Figure 2** shows that the mean eye height (dimension 2 in **Figure 1**) for men is 1650 mm (SD=74 mm) and for women the mean is 1537 mm (SD=65 mm).

Therefore for men any painting should be hung so that its centre is 1650 ± 74 mm above the floor. That is between 1576 mm and 1724 mm above the floor. For women the centre of the painting should be hung so that it is between 1472 mm and 1602 mm. To ensure that both men and women are satisfied the designer would be advised to hang each painting so that its centre is between 1576 mm and 1602 mm above the floor.

Other design problems require more complex mathematics. For example a shop fitting company might be asked to ensure that the top shelf in a supermarket should be placed so that it is only beyond the vertical grip reach of 10% of the population. In this case designers may assume that the distribution of values for body dimensions in a population can be modelled by a normal distribution. The mathematics associated with the normal distribution can then be used in the design process.

The Table in **Figure 2** shows that the mean vertical grip reach (dimension 6 in **Figure 1**) for men is 2090 mm (SD=107 mm) and for women the mean is 1901 mm (SD=80 mm).

In this shelf design problem the designer might choose to consider only women shoppers since they have a lower vertical reach height.

A standard z-value for women can be found using $z = \frac{x - 1901}{80}$

The problem then is to find a height so that only 10% of the population is at the lower end of the distribution as in **Figure 3**. Considering the symmetry of the distribution we need to find a value of *x* so that

$z = 1.28 = \frac{1901 - x}{80}$ leading to $x = 1799$ mm

Note that the value 1.28 can be found using tables, a calculator or from a summary of frequently used values such as given in the table in **Figure 4**.

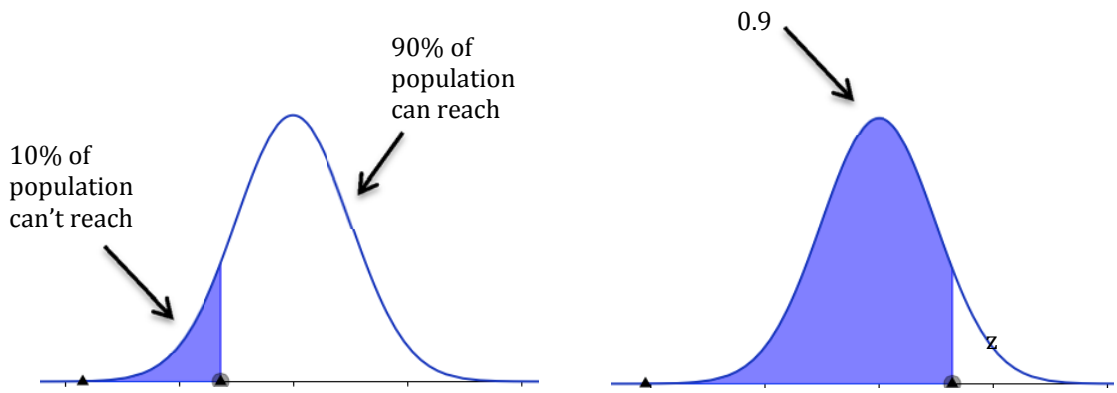

Figure 3. Distribution of vertical grip reach heights to access top shelf in a supermarket display.

Designers who carry out such calculations might use the table in **Figure 4** that quickly shows how to find values for commonly used percentiles of the population with mean μ and standard deviation σ

percentile	value		percentile	value
1st	$\mu - 2.33\sigma$		99th	$\mu + 2.33\sigma$
5th	$\mu - 1.65\sigma$		95th	$\mu + 1.65\sigma$
10th	$\mu - 1.28\sigma$		90th	$\mu + 1.28\sigma$
20th	$\mu - 0.84\sigma$		80th	$\mu + 0.84\sigma$
25th	$\mu - 0.67\sigma$		75th	$\mu + 0.67\sigma$

Figure 4. how to find values for commonly used percentiles of the population with mean μ and standard deviation σ

Of course when deciding the maximum height of the shelf the total population, including men, should have been taken into account. This requires finding values for the combined means and standard deviations. If for men the mean is μ_m and standard deviation σ_m and for women the mean is μ_w and standard deviation σ_w the combined mean, μ, and standard deviation , σ, are given by

$$\mu = \%\text{men} \times \mu_m + \%\text{women} \times \mu_w$$
$$\sigma^2 = \%\text{men} \times \sigma_m^2 + \%\text{women} \times \sigma_w^2 + \%\text{men} \times \%\text{women} \times \left(\mu_m - \mu_w\right)^2$$

where %men is the proportion of men in the population and %women the proportion of women.

If the proportion of men in the UK is 49%
Then, for vertical grip reach
$\mu = 1994$ mm and $\sigma = 133$ mm

This leads to a revised value for the maximum shelf height of 1824 mm

4) Earnings

The data below is edited from the Annual Survey of Hours and Earning for 2014 (UK) (http://www.ons.gov.uk/ons/dcp171778_385428.pdf)

Median full-time gross weekly earnings in current and constant (2014) prices, UK, April 2004 to 2014

£ per week

Year (April)	Current	Constant (2014 prices)	Current	Constant (2014 prices)	Current	Constant (2014 prices)	Change in constant prices (%)
2004	419.2	549.1					3.4
2005	431.2	554.0					0.9
2006	446.4	562.3	443.6	558.8			1.5
2007			457.6	560.9			0.4
2008			479.1	570.4			1.7
2009			488.5	568.3			-0.4
2010			498.5	559.2			-1.6
2011			500.7	537.6	498.3	535.0	-4
2012					506.1	527.5	-1.4
2013					517.4	526.4	-0.2
2014					518.0	518.0	-1.6

The table below shows how weekly earnings are distributed, giving the weekly earnings at the 10th, 25th, 50th, 75th and 90th percentiles.

Distribution of full-time gross weekly earnings, UK, April 2004 to 2014

£ per week

Year (April)	90th	75th	50th	25th	10th	90th	75th	50th	25th	10th	90th	75th	50th	25th	10th
2004	814.4	590.6	419.2	298.0	230.3										
2005	850.5	611.6	431.2	305.3	235.4										
2006	886.2	632.5	446.4	316.2	244.1	881.6	630.5	443.6	315.2	243.8					
2007						907.1	650.5	457.6	325.8	252.9					
2008						950.7	677.9	479.1	338.8	262.2					
2009						971.4	692.5	488.5	347.0	270.3					
2010						986.1	705.3	498.5	354.6	275.7					
2011						1,002.1	711.0	500.7	355.9	277.6	998.5	708.4	498.3	354.0	276.2
2012											1,001.9	714.8	506.1	360.0	282.1
2013											1,020.0	729.1	517.4	368.1	287.5
2014											1,024.4	732.7	518.0	369.9	287.9