Lesson 1: Working with matrix algebra

Suggested Teaching Time: 17 hours

Learning Outcome: 1. Be able to use matrix algebra to solve civil-engineering problems

Торіс	Suggested Teaching	Suggested Resources
AC 1.1 Perform operations in matrix algebra	 Delivery should include whole-class teaching to confirm the learners have with a thorough grounding in the matrix arithmetic processes, followed by individual practice in solving typical matrix problems. A number of points should be noted: Matrix notation: learners should use large round brackets instead of square ones, to avoid any confusion between the straight vertical lines denoting the matrix determinant Elements of some of the matrices to be worked with should be complex numbers Learners should spend time consolidating the techniques of Summation of matrices Subtraction of matrices by a scalar constant Finding the product of two or more matrices up to 3 by 3. Tutors should stress that, except in special cases, A x B ≠ B x A and learners should be able to demonstrate the standard proof. Transposing matrices Defining and using the identity or unit matrices I₂, I₃, I₄ etc. Calculating the determinants of a 2 by 2 matrix using A x A⁻¹ = I and by 	Book: Bird, J. O., <i>Higher</i> <i>Engineering Mathematics</i> 7 th edition (Routledge 2014), ISBN-13: 978-0415662826 Websites: <u>http://mathworld.wolfram.com/</u> <u>http://www.mathcentre.ac.uk/</u> General resource: free e- textbooks: <u>http://bookboon.com/</u>
AC 1.2 Evaluate the determinants of a matrix	 i. Interchanging the elements on the leading diagonal (\) ii. Changing the sign of the other two elements iii. Multiply the new matrix by the reciprocal of the determinant of the original matrix Working with 3 by 3 matrices, learners should be proficient in defining and calculating the Minor and cofactor: definition and how to obtain them Determinant: definition, process of calculation and simplification rules Adjoint: definition and how to obtain it 	



Lesson 1: Working with matrix algebra

Suggested Teaching Time: 17 hours

Learning Outcome: 1. Be able to use matrix algebra to solve civil-engineering problems

Торіс	Suggested Teaching	Suggested Resources
AC 1.4 Obtain the inverse of a square matrix	• Reciprocal using $A^{-1} = \frac{adj a}{ A }$ Having spent sufficient time practising the techniques of manipulating matrices, this powerful tool will be ready for use. Learners should now progress to the techniques of solving simultaneous equations using matrix methods. In particular, the following should be covered:	
AC 1.3 Solve simultaneous equations using Cramer's rule AC 1.5 Apply Gaussian elimination to solve simultaneous equations	 Solution by determinants Cramer's rule Gaussian elimination: systematic elimination by row transformations in the augmented matrix Eigenvalues and eigenvectors Also at this point, problems in a civil-engineering context appropriate to the learner's specialisation should be introduced covering subjects such as: Structural analysis (forces, vectors, mass, tension, loads etc.) 	
AC 1.6 Apply matrix algebra to solve civil- engineering problems described by sets of simultaneous equations	 Vibration analysis (coupled oscillations) Considerable time should be spent by the learner in exploring the types of civil-engineering problems which can solved using matrices 	



Lesson 2: Using vectors

Suggested Teaching Time: 10 hours

Learning Outcome: 2. Be able to use vectors methods to solve civil-engineering problems

Торіс	Suggested Teaching	Suggested Resources
AC 2.1 Perform operations with vectors	 Delivery is likely to be via whole-class teaching initially. However, the majority of learner activity will be in group and self-study. Tutors should ensure firstly that the learners have a good grounding in vector notation and vector resolution. There should be a period of revision of the following in an civil-engineering context: Scalar and vector definition Drawing vectors, including addition by both 'nose to tail' and parallelogram methods Finding resultants of two or more forces by resolution into horizontal and vertical 	Book: Bird, J. O., <i>Higher</i> <i>Engineering Mathematics</i> 7 th edition (Routledge 2014) ISBN-13: 978-0415662826 Website
AC 2.2 Solve civil- engineering problems using vectors.	 components Addition of vectors by calculation Subtraction of vectors Velocity problems i, j, k notation There should be a period of practice of a range of set civil-engineering problems involving the use of the above, until the learner feels confident in solving them. Subjects could include fluid or gas flow, stresses, strains and displacements in structures, and deformation of materials. This may only take a short time it the learner has recently progressed from a lower level of civil engineering mathematics. 	http://mathworld.wolfram.com/
	 The learner should then progress to vector and scalar products, with related civil-engineering problems being introduces as soon as is appropriate. Subjects and techniques which should be covered are: The unit vector and unit triad: definitions and calculations i.e.: the unit vector for oa being ^{oa}/_{oa} where oa is the vector and oa is the magnitude of the vector, and the unit triad being three unit vectors at right angles to each other 	



Lesson 2: Using vectors

Suggested Teaching Time: 10 hours

Learning Outcome: 2. Be able to use vectors methods to solve civil-engineering problems

Торіс	Suggested Teaching	Suggested Resources
	 The expression for the movement of an object in space from an origin o to a point r in space in terms of i, j, and k: or = xi +yj = zk The scalar or dot product of two vectors e.g.: oa · ob = oa · ob · cos(θ₂.θ₁) where θ₂ > θ₁ Vector or cross product: oa · ob = oa · ob · sinθ Vector equation of a line in Cartesian form Vector equation of a plane 	



Lesson 3: Introducing differentiation



Learning Outcome: 3. Be able to use differential and integral calculus to solve civil-engineering problems

4. Be able to apply numerical analy	ysis to solve civil-engineering problems
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Торіс	Suggested Teaching	Suggested Resources
AC 3.1 Determine first and higher order derivatives of functions	Delivery is likely to consist of a mixture of whole-class learning of a topic followed by a period of group work, backed up with individual practice. As with previous lessons, the examples used both in the teaching and the example problems should, as far as possible, be in a civil-engineering context in order to maintain focus. Differentiation should ideally be taught from first principles i.e.: the rate of change of a simple function within diminishing limits, and learners should be able to relate to variable quantities in civil engineering such as the rate of change of temperature in a room when controlled by air conditioning, the deflection of a structural beam under a dynamic load etc. This helps to give a visual focus	Book: Bird, J. O., <i>Higher</i> <i>Engineering Mathematics</i> 7 th edition (Routledge 2014) ISBN-13: 978-0415662826 Websites: http://mathworld.wolfram.com/
AC 3.2 Use differential calculus to solve problems	to return to whenever the learner encounters a challenging differential calculus problem. Often learners at this level will have been proficient in calculus in the past and will merely need revision to bring them back up to speed. In this case, the best approach is to practise solving problems in different contexts until they become second nature. Learners should be encouraged to commit as many of the standard derivatives as possible (see the data sheet from Lesson 2), particularly those for ax^n , $\sin ax$, $\cos ax$, e^{ax} and ln^{ax} . Learners should also be able to find higher derivatives $f''(x)$, $f'''(x)$ etc.at will. Appropriate standard notation should be used, but learners must be aware of the variations they will come across in literature and on websites etc. from different sources. Leibnitz notation is the most commonly used e.g.: the derivative or differential coefficient $\frac{dy}{dx}$ is the same as $f'(x)$. The important thing is to avoid confusion. The next stage is for the learners to master three important rules or methods of differentiating awkward functions. These are the product and quotient rules, and the chain rule, or 'function of a function'. When the learner has had plenty of practice in solving problems and is proficient in	http://www.mathcentre.ac.uk/



Lesson 3: Introducing differentiation

Learning Outcome: 3. Be able to use differential and integral calculus to solve civil-engineering problems

4. Be able to apply numerical analysis to solve civil-engineering problems
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Торіс	Suggested Teaching	Suggested Resources
	selecting and using the correct method they can progress to the practical applications of the methods of differentiation. They should by now have committed to memory the derivatives of $(ax \pm b)^n$, $\sin^n(ax \pm b)$, $\cos^n(ax \pm b)$, $e^{(ax \pm b)}$ and $ln^{(ax\pm b)}$. The first of these is the calculation of maxima, minima and points of inflexion of the graph of particular functions and indeed how to recognise whether or not the function has any or all of them. Here all of the practice problems can and should be in an engineering context. It will help the learner to not only use the techniques for a useful purpose, but to think of real engineering problems in terms of their methods of solution.	
	Finally learners will need to be able to differentiate implicit functions using the product and quotient rules, and logarithmic functions using the chain rule (function of a function) and natural logarithms to differentiate some complex functions. Examples of the complexity of practice problem are:	
	Differentiate with respect to x:	
	$y = \frac{e^{4x} \sin 2x}{\sqrt{x^2 + 4}}$	
	Also Evaluate $\frac{dy}{dx}$ when $x = 2$ for:	
	$y = \frac{(x-1)^2 \sqrt{(3x+1)}}{\sqrt{(x^2+7)^2}}$	
	Examples of contextualised revision questions are to be found in the recommended books.	



Lesson 4: Introducing integration

Suggested Teaching Time: 10 hours

Learning Outcome: 3. Be able to use calculus to solve civil-engineering problems

Торіс	Suggested Teaching	Suggested Resources
AC 3.3 Determine indefinite and definite integrals of algebraic and trigonometric functions and differential equations	Delivery is likely to consist of whole-class learning of a topic, followed by a period of group work where appropriate, and backed up with individual practice. It is likely that learners will need to do a considerable amount of self-study and practice in order to become proficient in the techniques in this lesson. The concept of integration as the reverse of differentiation, and also as a process of summation, is important and the learner must grasp the basic rules of these two processes. The meaning of the notation should be made clear also e.g.: $\int 2x dx + c$ means the integral of $2x$ with respect to x , and c denotes the possible presence of a constant. Also the standard integrals for common functions should be studied and as many as possible committed to memory, particularly the integrals of a constant, ax^n (where $n \neq 1$), $\frac{1}{x}$, $\sin(ax \pm b)$, $\cos(ax \pm b)$ and $e^{(ax\pm b)}$. The concept of limits and evaluating the integral between them should be learned and practised with all of the standard functions. The learner should also clearly understand the difference between an indefinite integral i.e. one containing the constant 'c' and a definite integral i.e. one in which limits have been applied. At this stage, engineering context can be introduced; the learner should be able to solve problems involving, for instance, entropy change, volume of gas or liquid in a container, or voltage waveforms. Learners should be able to find the mean and root mean square (RMS) values of a waveform. Having practised these integration methods and become competent in using them, the learner can now be introduced to some of the applications of integration, starting with finding the area under a curve, the centroid of a lamina and the volumes of solids of revolution of a plane area using the theorem of Pappus. From this the subject can be developed to include the second moment of area and moments of inertia since they are both normally designated by the letter <i>I</i> . Careful distinction should be made	Book: Bird, J. O., <i>Higher</i> <i>Engineering Mathematics</i> 7 th edition (Routledge 2014) ISBN-13: 978-0415662826 Websites: http://mathworld.wolfram.com/ http://www.mathcentre.ac.uk/



Lesson 4: Introducing integration

Suggested Teaching Time: 10 hours

Learning Outcome: 3. Be able to use calculus to solve civil-engineering problems

Торіс	Suggested Teaching	Suggested Resources
	between the two properties. Problems in a civil-engineering context can then be introduced covering, for instance, beam deflections, movement of structures in strong winds etc.	
AC 3.4 use integral calculus to solve problems	The next set of integration methods can now be introduced, using algebraic, trigonometric and hyperbolic substitution, and partial fractions. Learners should become proficient at integrating trigonometric functions including: $\sin^2 x$, $\cos^2 x$, $\tan^2 x$ and $\cot^2 x$; functions that have powers of sine and cosine and those that are products of sine and cosine. Learners should also be proficient in the use of the <i>sin</i> θ , $\cos \theta$, <i>sinh</i> θ and $\cosh \theta$ substitutions, and in integration by parts. Finally in this lesson learners should be able to apply integration methods to solve differential equations where the variables are separable. Learners should be able to do the following: identify and sketch families of curves, define first and second order equations, particular and general solutions, and boundary conditions. Problems for practice should relate to: • Applications of integration to areas • Volumes of revolution • Centres of mass • Moments of inertia • Second moments of area • Mean value and root mean square (RMS) values These should be in a civil-engineering context.	





Lesson 5: Numerical methods in calculus

Suggested Teaching Time: 20 hours

Торіс	Suggested Teaching	Suggested Resources
AC 4.1 Use numerical	Delivery is likely to consist of whole-class teaching for each topic interspersed with periods	Book:
find the roots of a function	of individual work on set problems. The learner should now progress to numerical iterative methods of solving equations, starting with the bisection method from first principles and the use of tabular data to find the roots.	Bird, J. O., <i>Higher</i> <i>Engineering Mathematics</i> 7 th edition (Routledge 2014)
	Secondly the secant method should be taught:	ISBN-13: 978-0415662826
	$x_{n-1} - x_{n-2}$ $x_{n-2}f(x_{n-1}) - x_{n-1}f(x_{n-2})$	Websites:
	$x_n = x_{n-1} - f(x_{n-1}) \frac{f(x_{n-1}) - f(x_{n-2})}{f(x_{n-1}) - f(x_{n-2})} = \frac{f(x_{n-1}) - f(x_{n-2})}{f(x_{n-1}) - f(x_{n-2})}$	http://mathworld.wolfram.com/
	Using initial values x_0 and x_1 close to the root.	http://www.mathcentre.ac.uk/
	Third to be introduced is the Newton-Raphson, or Newton Method:	
	If r_1 is the approximate value of a real root of the equation $f(x) = 0$ then a closer approximation to the root r_2 is given by:	
	$r_2 = r_1 - \frac{f(r_1)}{f'(r_1)}$	
	Problems in a civil-engineering context should be worked through to consolidate the learning. Subjects could include analysis of waveforms, oscillations and vibrations in beams and other structural members.	
AC 4.2 Apply numerical methods of integration to engineering variables	Numerical methods of integration are important and the learner should be able to apply two rules: the Trapezoidal rule and Simpson's rule:	
	The Trapezoidal rule: for those learners who are unfamiliar with the use of this rule, it is	
	best to start with the arithmetical method of finding the area under an irregular curve.	

Lesson 5: Numerical methods in calculus

Торіс	Suggested Teaching	Suggested Resources
	Dividing the area between the curve and the x - y axes into a number of thin strips, each of which approximates to a trapezoid. The area bounded by the curve and the x - y axes is given by :	
	(Interval width) [½ (first + last ordinate) + sum of remaining ordinates]	
	Quickly progressing then to numerical integration, the learner will see that the above is equal to $\int_{a}^{b} y dx$. Learners should be able to write down the standard proof of this from memory, and apply the rule to any problem. This applies also to the related Mid-ordinate	
	rule	
	Simpson's rule: follows on from the preceding two but is a more accurate approximation; learners should always be encouraged to think in terms of errors, how they can be reduced in their calculations and how they affect the calculation if they can't be eliminated. Learners should be able to demonstrate how the rule works and how it is derived, including the fact that it only works with an even number of intervals (odd number of ordinates)	
	Practice problems here should focus on	
AC 4.3 Apply numerical methods for the solution	 Areas and volumes Centres of gravity Moments of inertia Second moment of area 	
of ordinary differential equation models	Taylor's series may be better understood if the derivation of Maclaurin's series is understood first:	



Lesson 5: Numerical methods in calculus



Торіс	Suggested Teaching	Suggested Resources
	From the power series of $f(x)$ assumed as $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \dots$	
	Where <i>a₀, a₁, a₂</i> are constants	
	Arriving at $f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots$	
	The conditions of the Maclaurin's series should also be covered, as should numerical integration limiting values and L'Hôpital's rule for functional evaluation:	
	$\lim_{x \to a} \left(\frac{f(x)}{g(x)} \right) = \lim_{x \to a} \left(\frac{f'(x)}{g'(x)} \right) \text{ provided } g'(a) \neq 0$	
	Typical civil-engineering problems using the above should be practised by learners. These are likely to include problems on beam flexure and the derivation of the elastic curve of a transversely loaded beam.	
	Taylor's series should then be derived from Maclaurin's series:	
	$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \dots$	
	Learners should be able to use series expansions to obtain approximations of the functions e^x , sin x, cos x and tan x.	
	Civil engineering-based problems should be practised by the learners, in a range of contexts, which could include fluid flow rates, vibration, distortion of a body or structure under its own weight, spread of a contaminant in a water course or fluid system. In fact any physical property where data are only available at discrete points or intervals.	
	Work with differential equations may be revision for the members of the class, but, as always, there will be those who have not remembered the rules and methods as well as the others. So it is often helpful to revise the families of curves, the notation and the basic rules of differentiation and solving differential equations by separation of variables. Learners should be able to recognise and to readily solve first order differential equations	



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Lesson 5: Numerical methods in calculus

Suggested Teaching Time: 20 hours

Торіс	Suggested Teaching	Suggested Resources
	of the following forms:	
	• $\frac{dy}{dx} = f(x)$	
	• $\frac{dy}{dx} = f(y)$	
	• $\frac{dy}{dx} = f(x) \cdot f(y)$	
	Problems in a civil-engineering context could be introduced here.	
	Solving exact first order differential equations: learners should be able to define and solve exact first order differential equations by partial differentiation, and should also be able to test a given equation for exactness.	
	Solving linear first order differential equations: learners should be able to define and	
	recognise a first order linear differential equation and to solve, using an integrating factor, equations of the form:	
	$rac{dy}{dx}+ \ oldsymbol{P}oldsymbol{y}=oldsymbol{Q}$ where P and Q are functions of x only	
	Learners should practise solving problems in a civil-engineering context, which could	
	include: equations of motion of particles in a resisting fluid, alternating current, impurities in an engine oil tank etc.	
	The next topic in this part of the unit involves numerical methods for solving ordinary and	
	partial differential equation models of civil engineering systems. Examples should cover a range of relevant systems Learners should approach each method one by one, where	
	appropriate comparing the results and procedures for individual equations.	
	Starting with ordinary differential equations, the contextual problems should centre	
	particularly around areas, volumes, centres of gravity and moments of inertia. Methods to	
	be addressed by the learners are:	
	Taylor's series	

Lesson 5: Numerical methods in calculus

Suggested Teaching Time: 20 hours



Торіс	Suggested Teaching	Suggested Resources
	• Euler, taking the first two terms of the Taylor series: $f(a + h) = f(a) + hf'(a)$	
	• Euler-Cauchy: learners should be able to demonstrate the difference in accuracy between this and the standard Euler method	
	 Runge-Kutta: learners should not be expected to derive or prove this method, however they should be able to follow the seven steps in order to arrive at a highly accurate solution to an equation. 	
	Finite difference: forward, backward and central	
4.4 Apply iterative numerical methods to the solution of partial differential equation models	It would be a useful exercise for learners to construct a table of solutions for a particular equation, say $y = x + 1 + e^x$ using each of the above methods to compare their accuracies. This will help the learner to remember the relative accuracies and to be able to select an appropriate method for a particular problem.	
	Lastly solution of partial differential equation models of civil engineering systems using iterative numerical methods. The following should be included:	
	Finite difference methods (similar to above)	
	 The Jacobi¹ iterative method for a square set of linear equations: Ax = b 	
	• The Gauss-Seidel ¹ iterative method for a square set of linear equations: Ax = b	
	Learners should then complete a series of problems based on typical and relevant	
	civil engineering systems. The sim is to senselidete learning for Learning Outcomes 1.4	
	and to give learners practice in selecting and using methods taught in these LOs.	

¹ These two methods do not appear in the recommended textbook, but can be found on the Wolfram.com website.







Lesson 6: Statistical distribution

Suggested Teaching Time: 8 hours

Learning Outcome: 5. Understand forms of statistical distribution

Торіс	Suggested Teaching	Suggested Resources
AC 5.1 Explain the uses of statistical uniform distribution	Prior to starting the teaching for the final three learning outcomes in this unit, it would be useful to	Book:
	 Confirm that learners have at least a good appreciation of the following: How to present statistical data Mean, median, mode and standard deviation 	Bird, J. O., <i>Higher</i> <i>Engineering Mathematics</i> 7 th edition (Routledge 2014)
	Probability	ISBN-13: 978-0415662826
	Some additional teaching may be required if this is not the case. This will allow a solid grounding to begin studying Distributions .	Websites:
		http://mathworld.wolfram.com
	Delivery is likely to be mainly whole-class teaching with group work to reinforce the learning and facilitate more extensive statistical work.	http://www.mathcentre.ac.uk/
	Learners should be able to define and give examples of the use and characteristics of each of the distribution types listed:	(Tutors and learners might find this useful)
		http://cast.massey.ac.nz/
	Probability Binomial	Some useful civil engineering scenarios can be found here:
	Poisson	http://www.csupomona.edu/~
	Population	rosenkrantz/prinfo/civileng.ht
	NormalLinear correlation	
	Sampling	
	Student's <i>t</i> for small samples	

Lesson 6: Statistical distribution

Suggested Teaching Time: 8 hours

Learning Outcome: 5. Understand forms of statistical distribution

Торіс	Suggested Teaching	Suggested Resources
AC 5.2 Explain the concept of degrees of freedom.	Chi-squared Continuous Probability	
	 U-quadratic distribution, F-distribution for analysis of variance 	
	There is no particular order to teach these distribution types, however the above ordered list does offer a logical progression from one to another. Learners should be able to:	
	 Sketch typical distribution graphs Identify and define skews Be able to use the data tables associated with each distribution type Work with representative data sets from civil engineering sources and produce working graphs, probability data and analysis 	
	Degrees of Freedom: learners should have an overview of sampling and estimation theory, and in particular should be able to demonstrate how to estimate the mean of a population based on a small sample size. In the course of the latter, the learner will be able to explain the concept of the term 'degrees of freedom' .	



Lesson 7: Lesson Title

Suggested Teaching Time: 8 hours (Minimum)

Learning Outcome: 6. be able to use statistical tests

Торіс	Suggested Teaching	Suggested Resources
AC 6.1 Perform statistical tests of difference AC 6.2 Perform statistical tests of correlation	This lesson could be taught separately from the previous one, however tutors may prefer to integrate the two. Delivery is likely to be whole-class with group and/or individual work. It is unlikely that learners will have time to gather suitable statistical data to work with, so tutors are likely to have to provide carefully chosen data sets for learners to work with. Data sets should, wherever possible, be in a civil-engineering context. Statistical tests of difference	Book: Bird, J. O., <i>Higher</i> <i>Engineering Mathematics</i> 7 th edition (Routledge 2014) ISBN-13: 978-0415662826
	 Student-t for small samples Chi squared ANOVA (analysis of variance)¹ ANOVA, being a family of statistical tests, has the potential to become very complex very quickly. Tutors should consider both the available time and the relevance of the more complex forms. Comparison between ANANOVA and the t-test results and ease of working for one-way (single factor) analysis should be sufficient, however individual learners might wish to go further. 	Websites: http://mathworld.wolfram.com / http://www.mathcentre.ac.uk/ http://www.statstutor.ac.uk/to pics/ http://cast.massey.ac.nz/colle
³ An evolution	As above, students need to develop a sense of the differences and similarities between the more widely used tests, to be able to select the most appropriate one and to use it: Pearson's product-moment Spearman's rank Kendall tau rank Goodman and Kruskal's gamma ³	ction_public.htm http://www.csupomona.edu/~ rosenkrantz/prinfo/civileng.ht m#traffic



Lesson 8: Linear regression

Suggested Teaching Time: 8 hours (minimum)

Learning Outcome: 7. Be able to use linear regression techniques

Торіс	Suggested Teaching	Suggested Resources
TopicAC 7.1 Performlinear regressiontechniquesAC 7.2 makepredictions usinglinear regressiontechniques.	Suggested Teaching Delivery is likely to include group and individual work on example datasets, following a period of whole-class teaching. Tutors should prepare suitable sets of relevant data for learners to work with, which should include questions that might need answering before a project or task would be allowed to proceed, Learners should be familiar with the importance of the accuracy of their analysis and predictions in the decision-making process. Data sets should be of the following types, and where appropriate, learners should be able to recognise the type of data, e.g.: Linear and curvilinear Univariate Bivariate Multivariate	Suggested Resources Book: Bird, J. O., Higher Engineering Mathematics 7 th edition (Routledge 2014) ISBN-13: 978-0415662826 Websites: http://mathworld.wolfram.com / http://www.mathcentre.ac.uk/ http://www.statstutor.ac.uk/to pics/ http://cast.massey.ac.nz/coll ection_public.html http://www.csupomona.edu/~ rosenkrantz/prinfo/civileng.ht m#traffic